Chemical freezeout in relativistic $A+A$ collisions: is it close to the QGP?

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Abstract

Preliminary experimental data for particle number ratios in the collisions of Au+Au at the BNL AGS (11A GeV/c) and Pb+Pb at the CERN SPS (160A GeV/c) are analyzed in a thermodynamically consistent hadron gas model with excluded volume. Large values of temperature, T = 140–185 MeV, and baryonic chemical potential, μ_b = 590–270 MeV, close to the boundary of the quark-gluon plasma phase are found from fitting the data. This seems to indicate that the energy density at the chemical freezeout is tremendous which would be indeed the case for the point-like hadrons. However, a self-consistent treatment of the van der Waals excluded volume reveals much smaller energy densities which are very far below a lowest limit estimate of the quark-gluon plasma energy density.

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The quantitative results of quantum chromodynamics for the equation of state of strongly interacting matter have been obtained mostly in the high energy density region, $\varepsilon > 2$ GeV/fm^3, where one expects a weakly interacting gas of quarks and gluons — the quark-gluon plasma (QGP). The region of small ε corresponds to the hadron gas (HG) phase and can only be treated today in terms of the hadron degrees of freedom. Assuming a local thermodynamical equilibrium at the final freeze-out stage of nucleus-nucleus (A+A) collisions, one can estimate the particle number ratios without detailed knowledge of the complicated system evolution. Hadron abundances and ratios have been suggested as possible signatures for exotic states and phase transitions in dense nuclear matter (see, e.g., Refs. [1–5]).

Preliminary data for A+A collisions with truly heavy beams have recently become available: Au+Au at 11A GeV/c at the BNL AGS and Pb+Pb at 160A GeV/c at the CERN SPS [6]. A systematic analysis of these data could yield clues to whether a short-lived QGP phase with high ε exists during the hot and dense stage of these reactions. Recent fits of particle number ratios [7–12] at AGS and SPS energies in the framework of "thermal model" have led to large values of temperature, T, and baryonic chemical potential, μ_b, close to the boundary of the QGP phase [11,12]. The aim of the present paper is two-fold. First, we clarify the notion of the "thermal model" used in the literature. Second, we present our results for Au+Au (AGS) and Pb+Pb (SPS) collisions and examine whether the chemical freeze-out states are really close to the QGP. We find large values of the chemical freeze-out parameters T = 140–185 MeV and μ_b = 590–270 MeV. The values of the energy density ε and baryonic number density n_b are, however, strongly dependent on the specific thermal model formulation. In the ideal HG, the chemical freeze-out values of ε and n_b for SPS A+A collisions are indeed close to the QGP estimates. These values become, however, much smaller in thermodynamically consistent HG model with excluded volume.

The name "thermal model" has always been used in the literature when calculations of particle number ratios in A+A collisions are done with parameters T and μ_b. We stress, however, that different people used, in fact, very different "thermal models". It seems

natural to use the ideal HG as a thermal model at the freeze-out stage. However, such an ideal gas model becomes inadequate in high-energy A+A collisions. The chemical freeze-out parameters T and μ_b obtained from fitting the particle number ratios at AGS and especially SPS energies lead to artificially high particle number densities which contradict the assumed picture of non-interacting hadrons. The van der Waals (VDW) excluded volume procedure has recently been used in a number of thermal model calculations. We follow the excluded volume procedure of Ref. [13]. The "thermal models" of Refs. [7–12] also include VDW "corrections", but in *ad hoc* thermodynamically inconsistent ways. Besides, in the formulation of Refs. [7–10], particle ratios always remain the same as in the ideal gas for any choice of proper hadron volumes.

For a fixed particle number N, the VDW excluded volume procedure amounts to the substitution of the volume V by $V - vN$, where v is the parameter corresponding to the proper volume of the particle. Note that this VDW procedure, interpreted in statistical mechanics as an approximation to the gas of hard-sphere particles with radius r, requires that the volume parameter v is equal to the "hard-core particle volume", $\frac{4}{3}\pi r^3$, multiplied by a factor of 4 [14]. An extension of the excluded volume procedure for several particle species $i = 1, ..., h$, has been done with the bf ansatz of the substitution $V \rightarrow V - \sum_{i=1}^{h} v_i N_i$. The pressure function in the grand canonical formulation is given then by the equation [13]:

$$p(T, \mu_1, ..., \mu_h) \;=\; \sum_{i=1}^{h} p_i^{id}(T,\; \mu_i \;-\; v_i\; p(T, \mu_1, ..., \mu_h)) \;, \tag{1}$$

where p_i^{id} is the ideal gas pressure of i-th particle

$$p_i^{id}(T, \mu_i) \;=\; \frac{d_i}{6\pi^2} \int_0^\infty k^2 dk \; \frac{k^2}{(k^2 + m_i^2)^{1/2}} \left[\exp\left(\frac{(k^2 + m_i^2)^{1/2} - \mu_i}{T} \right) \pm 1 \right]^{-1} \;,$$

with d_i the number of internal degrees of freedom (degeneracy), μ_i the particle chemical potential and m_i the mass. Particle number density for the i-th species is calculated from Eq. (1) as

$$n_i \;\equiv\; \frac{\partial p}{\partial \mu_i} = \frac{n_i^{id}(T, \tilde{\mu}_i)}{1 \;+\; \sum_{j=1}^{h} v_j \; n_j^{id}(T, \tilde{\mu}_i)} \;, \tag{2}$$

with

$$\tilde{\mu}_i \;\equiv\; \mu_i \;-\; v_i\, p(T,\mu_1,...,\mu_h)\;, \quad i=1,...,h\;. \tag{3}$$

The energy density of the HG with VDW repulsion is

$$\varepsilon \;=\; \frac{\sum_{i=1}^{h}\varepsilon_i^{id}(T,\tilde{\mu}_i)}{1\;+\;\sum_{j=1}^{h} v_j\; n_j^{id}(T,\tilde{\mu}_j)}\;, \tag{4}$$

where n_i^{id}, ε_i^{id} are the ideal gas quantities. The VDW repulsion leads to strong suppression of particle number densities and therefore all thermodynamical functions in the HG with the VDW repulsion become much *smaller* than those in the ideal gas at the same T and $\mu_1,...,\mu_h$ (see Ref. [15] for details).

Particle chemical potentials μ_i regulate the values of conserved charges. For simplicity we neglect the effects of non-zero electrical chemical potential which were considered in Ref. [16,17]. The chemical potential of the i-th particle can then be written as

$$\mu_i \;=\; b_i\,\mu_b \;+\; s_i\,\mu_s\;, \tag{5}$$

in terms of baryonic chemical potential μ_b and strange chemical potential μ_s, where b_i and s_i are the corresponding baryonic number and strangeness of the i-th particle. The hadronic gas state is defined by two independent thermodynamical parameters, T and μ_b. The strange chemical potential $\mu_s(T,\mu_b)$ is determined from the requirement of zero strangeness

$$n_s(T,\mu_b,\mu_s) \;\equiv\; \sum_{i=1}^{h} s_i n_i(T,\mu_i) \;=\; 0\;. \tag{6}$$

For the SPS A+A data it has been observed in Refs. [8,9] that their "thermal model" (for particle number ratios it is, in fact, equivalent to the ideal gas model) is unable, within a single set of freeze-out parameters, to reproduce simultaneously the strange particles and anti-baryon to baryon ratios together with ratios where pions are involved. Experimental pion to nucleon ratio and ratios of pions to other hadrons are larger than the ideal gas predictions. Several mechanisms, including the chemical non-equilibrium with $\mu_\pi > 0$ in the pion subsystem and the possibility of straight QGP hadronization [8,9,18] were proposed, but

no satisfactory solution has been found. Another possibility with an incomplete strangeness chemical equilibrium and parameter $\gamma_s < 1$ has been recently studied in Ref. [19]. We follow the HG model with complete chemical equilibrium. To enlarge the number of pions in our VDW HG we assume that the pion proper volume parameter v_π is smaller than the proper volume parameters for other hadrons which we put to be equal to v. In the case where not all of the v_i's are equal, the hadron volume parameters v_i do influence the particle number ratios through the modification of Eq. (3) in the particle chemical potentials, required by thermodynamical self-consistency. The effect is quite evident: hadrons which take up less space (i.e., smaller values of v_i) are preferably produced. The particle number ratios of those small hadrons to larger ones increase as compared with the ideal gas results. We assume different hard-core radii: r_π for pions and r for all other hadrons ($r > r_\pi$), with $v_i = 4\cdot\frac{4\pi}{3}\, r_i^3$.

Baryon and meson resonances are of great importance for the measured particle number ratios at AGS and SPS energies. All known resonance states with masses up to 2 GeV are included in our calculations with subsequent resonance decays to stable hadrons. We use the compilation of experimental data for particle number ratios in Au+Au at the BNL AGS ($11A$ GeV/c) and Pb+Pb at the CERN SPS ($160A$ GeV/c) which were presented by Stachel at QM'96 (see Ref. [12] and references therein). Our fits within the VDW HG model is shown in Fig. 1 (see also [15]). From fitting the data we found the following model parameters:

$$\mathrm{AGS:}\quad T \;\cong\; 140\ \mathrm{MeV}\ ,\quad \mu_b \;\cong\; 590\ \mathrm{MeV}\ , \tag{7}$$

$$\mathrm{SPS:}\quad T \;\cong\; 185\ \mathrm{MeV}\ ,\quad \mu_b \;\cong\; 270\ \mathrm{MeV}\ . \tag{8}$$

There are a number sets of possible values for hard-core radii which give the same hadron ratios in Fig. 1. If we put $r_\pi = 0$, then one finds $r = 0.50$ fm and $r = 0.46$ fm for AGS and SPS cases, respectively. On the other hand, the AGS and SPS data in Fig. 1 can be fitted simultaneously with following set of

$$r_\pi \;\cong\; 0.62\ \mathrm{fm}\ ,\qquad r \;\cong\; 0.8\ \mathrm{fm}\ . \tag{9}$$

Due to $r_\pi < r$ the ratios of *direct* pions to other hadrons increase by a factor of 2.0 for AGS (7) and 2.6 for SPS (8) in comparison to the case $r_\pi = r$ (in this later case all ratios are very close to their ideal gas values). It results in the increase of *total* pion to other hadron ratios about 25% for AGS (7) and 33% for SPS (8).

The calculated energy density ε for the above T, μ_b values (7,8) are

$$\text{AGS}: \quad \varepsilon^{id} \;\cong\; 0.72 \text{ GeV/fm}^3 \;, \quad \varepsilon^{VDW} \;=\; 0.25 - 0.08 \text{ GeV/fm}^3 \;, \tag{10}$$

$$\text{SPS}: \quad \varepsilon^{id} \;\cong\; 1.58 \text{ GeV/fm}^3 \;, \quad \varepsilon^{VDW} \;=\; 0.43 - 0.10 \text{ GeV/fm}^3 \;, \tag{11}$$

for the ideal HG (i.e. $r_\pi = r = 0$) and VDW HG, respectively. The corresponding values for the baryonic number density n_b are

$$\text{AGS}: \quad n_b^{id} \;\cong\; 0.40 \text{ fm}^{-3} \;, \quad n_b^{VDW} \;=\; 0.13 - 0.04 \text{ fm}^{-3} \;, \tag{12}$$

$$\text{SPS}: \quad n_b^{id} \;\cong\; 0.40 \text{ fm}^{-3} \;, \quad n_b^{VDW} \;=\; 0.10 - 0.04 \text{ fm}^{-3} \;. \tag{13}$$

The higher values of ε^{VDW} and n_b^{VDW} in the above equations are obtained for $r_\pi = 0$, $r = 0.50$ fm (AGS) and $r_\pi = 0$, $r = 0.46$ fm (SPS) while lower ones are found with parameters of Eq. (9) which fit simultaneously the AGS and SPS data. Note also that Eq. (3) gives an additional suppression by a factor of $\exp(-\, v_i\; p/T)$ for all particle number and energy densities of i-th particle in comparison to the inconsistent VDW gas treatment [7–10]. In the region of different T, μ_b, r_π and r considered above it leads to a suppression factor between 2 and 6 for the total energy density and baryonic number density.

To compare the energy densities of HG (10,11) with those of the QGP we use equation of state of the bag model, i.e., an ideal gas of u, d, s quarks, antiquarks and gluons with nonperturbative "vacuum pressure", $B \;\cong\; 400$ MeV fm^{-3} (see, e.g. [20]). The chemical potentials of quarks and antiquarks are given by the general equation (5) with $b_i = 1/3$ $(-1/3)$ for all quarks (antiquarks), $s_i = 0$ for u, d quarks and antiquarks, $s_i = -1$ $(+1)$ for s quark (antiquark). The consequence of zero strangeness requirement (6) is quite simple in the

QGP. It leads to equal values for the chemical potentials of s and $\overline{s}$ and hence both should be equal to zero. Thermodynamical functions within zero quark mass approximation can be easily calculated:

$$p^Q \;=\; \frac{\pi^2}{90}\,\frac{95}{2}\,T^4 \;+\; \frac{1}{9}\,T^2\,\mu_b^2 \;+\; \frac{1}{162\pi^2}\,\mu_b^4 \;-\; B\;, \tag{14}$$

$$\varepsilon^Q \;=\; \frac{\pi^2}{30}\,\frac{95}{2}\,T^4 \;+\; \frac{1}{3}\,T^2\,\mu_b^2 \;+\; \frac{1}{54\pi^2}\,\mu_b^4 \;+\; B\;, \tag{15}$$

$$n_b^Q \;=\; \frac{2}{9}\,\mu_b\,T^2 \;+\; \frac{2}{81\pi^2}\,\mu_b^3. \tag{16}$$

The lowest estimate of the energy density for the QGP can be obtained by putting

$$p^Q(T,\mu_b) \;=\; 0\;. \tag{17}$$

which defines a curve in the μ_b-T plane as shown in Fig. 2. The values of ε^Q (15) ($\varepsilon^Q = 4B$) and n_b^Q (16) along this curve are shown in Fig. 3. Note that the curve in the μ_b-T plane in Fig. 2 is not realistic for the HG-QGP phase transition. This is just a lower estimate for the μ_b-T boundary of such a transition. The values of T and μ_b given in Eqs. (7,8) are also shown as points in Fig. 2: AGS (7) square-point is very close to the QGP boundary and SPS (8) circle-point is in the "QGP phase". Our Fig. 2 is similar to the corresponding figures in Refs. [11,12]. To compare the energy densities of HG and QGP states one should remember that the energy density values are very different in the ideal HG and VDW HG. The ideal HG model was used recently to describe particle number ratios in AGS [17] and SPS [19] A+A collisions. Note that large values of $T \cong 193$ MeV and $\mu_b \cong 234$ MeV, close to our estimates (8), were found in Ref. [19] for Pb+Pb (SPS). Fig. 3 shows that ideal HG value of ε^{id} for the SPS parameters (8) is indeed rather close to the QGP phase. However, the VDW HG values ε^{VDW} for the same T and μ_b are very much smaller than ε^{id}. Ideal HG and VDW HG models lead to very different pictures of the chemical freezeout for A+A collisions at SPS energies: $\varepsilon^{id} \cong \varepsilon^Q$, but $\varepsilon^{VDW} << \varepsilon^Q$ and $n_b^{VDW} << n_b^Q$. We stress that very large values of ε^{id} and n_b^{id} are in an inevitable logic contradiction with the assumed picture of non-interacting hadrons.

In summary, we have analyzed the preliminary data for particle number ratios measured in the collisions of Au+Au at the BNL AGS ($11A$ GeV/c) and Pb+Pb at the CERN SPS ($160A$ GeV/c) with a thermodynamically consistent hadron gas model with excluded volume. Even though large values of temperature, T = 140–185 MeV, and baryonic chemical potential, μ_b = 590–270 MeV, close to the boundary of the quark-gluon plasma phase are found from fitting the data, the energy densities obtained always lie far below a lowest limit estimate of the QGP energy density because of the VDW suppression effects. Such a picture is consistent with a large energy density change during a HG-QGP phase transition.

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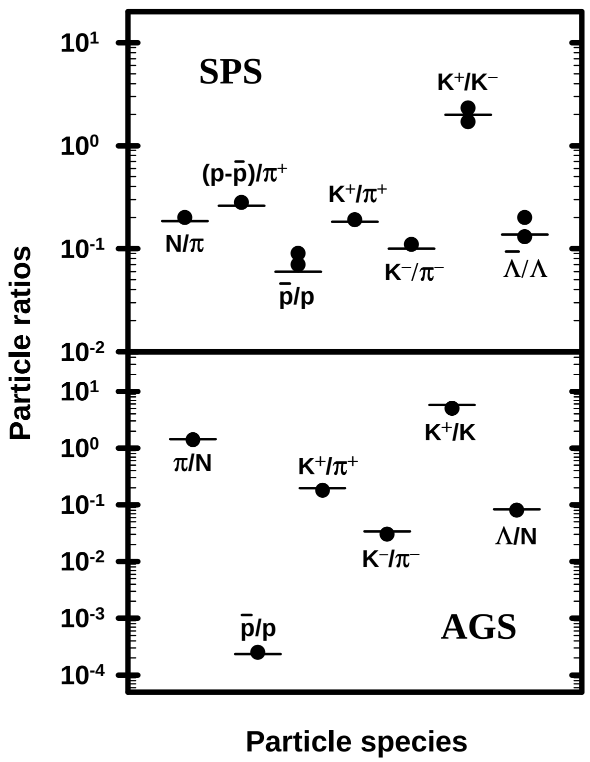
FIGURES

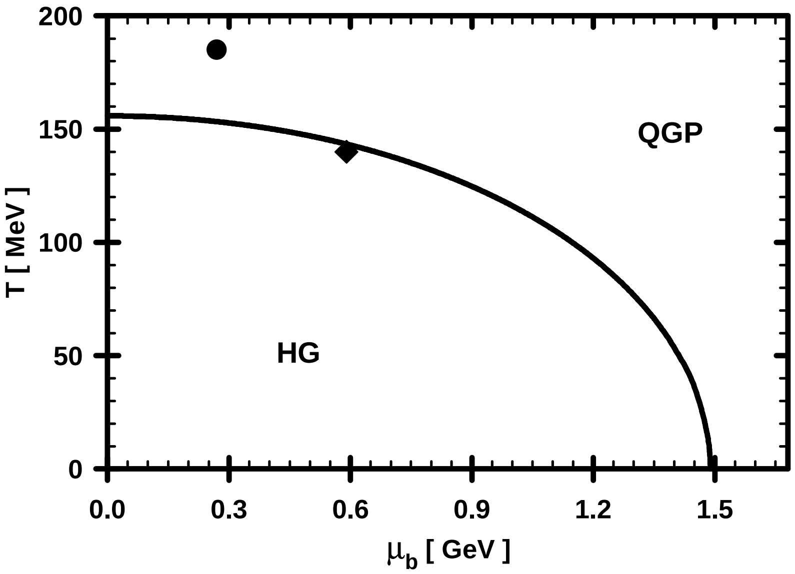
Fig. 1: Points are the preliminary experimental data for the particle number ratios (see Ref. [12] and references therein) for Au+Au AGS and Pb+Pb SPS collisions (in the lower and upper part of the figure respectively). The short horizontal lines are the model fit with $T \cong 140$ MeV, $\mu_b \cong 590$ MeV (AGS), and $T \cong 185$ MeV, $\mu_b \cong 270$ MeV (SPS). In both cases $r_\pi = 0.62$ fm, $r = 0.8$ fm.

Fig. 2: The solid line is the curve $p^Q(T, \mu_b) = 0$ (17) in the μ_b-T plane. The square and circle are the chemical freeze-out points of (7) and (8), respectively.

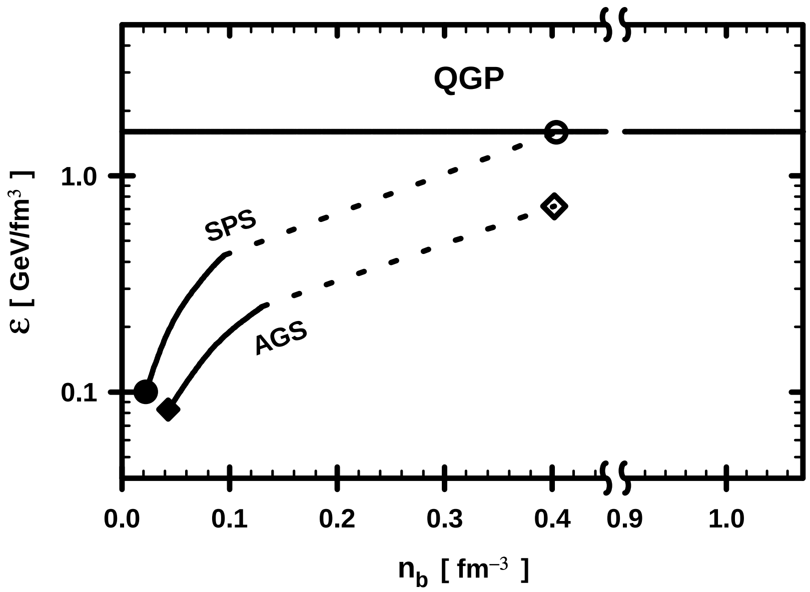
Fig. 3: The straight solid line gives the lowest energy density estimate, $\varepsilon^Q = 4B \cong 1.6$ GeV/fm^3, for the QGP. It corresponds to the curve $p^Q(T, \mu_b) = 0$ (17) of Fig. 2 in the n_b-ε plane. The largest value in the x-axis, $n_b \cong 1.08$ fm^{-3}, corresponds to the point $T = 0$ of the curve in Fig. 2. The open square and circle show the ideal HG values of ε^{id}, n_b^{id} for AGS (10),(12) and SPS (11),(13). The solid lines show the sets of the VDW HG values ε^{VDW}, n_b^{VDW} for AGS and SPS. The solid square and circle are the VDW HG values at AGS (7) and SPS (8) with hard-core radii (9).



SPS
AGS
K+/K−
(p-p̄)/π+
K+/π+
N/π
K−/π−
p̄/p
Λ̄/Λ
π/N
K+/K
Λ/N
10^1
10^0
10^-1
10^-2
10^-3
10^-4
Particle ratios
Particle species



200
150
100
50
0
T [MeV]
0.0
0.3
0.6
0.9
1.2
1.5
μ_b [GeV]
QGP
HG



QGP
SPS
AGS
1.0
0.1
ε [GeV/fm3]
0.0
0.1
0.2
0.3
0.4
0.9
1.0
nb [fm−3]